ENERGY PRINCIPLE

Steady Flow Energy Equation

For steady flow, the flow accumulation equal zero and if the **velocity distribution is constant**, then

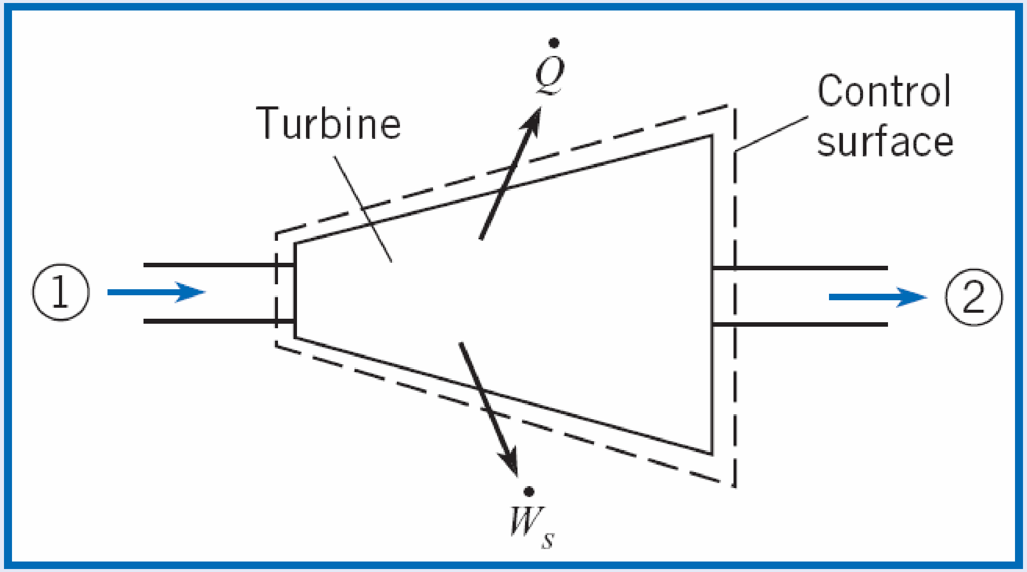
the Eqn. $\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{CV}(u + \frac{v^2}{2} + gz)\rho dQ + \int_{CS}(h + \frac{v^2}{2} + gz)\rho V \bullet dA$ becomes

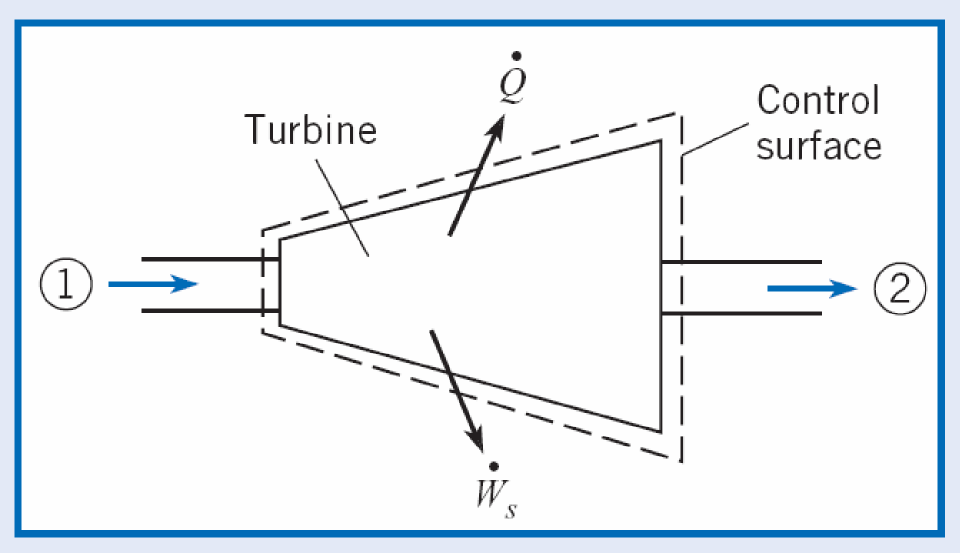
$$\dot{Q} - \dot{W}_S = \sum_{CS}\dot{m}_{out}\left(\frac{V^2}{2} + gz + h\right)_{out} - \sum_{CS}\dot{m}_{in}\left(\frac{V^2}{2} + gz + h\right)_{in}$$

Example (7.1)

A steam turbine receives superheated steam at 1.4 MPa absolute and 400°C, which corresponds to a specific enthalpy of 3121 kJ/kg. The steam leaves the turbine at 101 kPa absolute and 100°C, for which the specific enthalpy is 2676 kJ/kg. The steam enters the turbine at 15 m/s and exits at 60 m/s. The elevation difference between entry and exit ports is negligible. The heat lost through the turbine wall is 7600 kJ/h. Calculate the power output if the mass flow through the turbine is 0.5 kg/s.

Solution First sketch the general layout of the turbine, indicating the inlet and outlet velocities, shaft work, and heat transfer, as shown.





$$Given: h_1 = 3121\frac{kJ}{kg}, \quad h_2 = 2676\frac{kJ}{kg}, \quad V_{in} = 15\frac{m}{s}, \quad V_{out} = \frac{m}{s}, \quad \dot{Q} = 7600\frac{kJ}{kg}, \quad z_1 = z_2,$$

$$\dot{m} = 0.5\frac{kg}{\text{sec}}$$

Find: $\dot{W}_{Shaft}$?

Applying the steady flow energy equation

$$\dot{Q} - \dot{W}_S = \sum_{CS} \dot{m}_{out}\left(\frac{V^2}{2} + gz + h\right)_{out} - \sum_{CS} \dot{m}_{in}\left(\frac{V^2}{2} gz + h\right)_{in}$$

$$(gz)_{out} = (gz)_{in} = 0, \quad \dot{m}_{in} = \dot{m}_{out} = \dot{m},$$

$$\dot{W}_S = \dot{Q} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2}{2} - \frac{V_1^2}{2}\right)$$

$$\dot{W}_s = \frac{-7600}{3600}\frac{\text{kJ}}{\text{h}}\frac{\text{h}}{\text{s}} + 0.5\frac{\text{kg}}{\text{s}}\left[\frac{15^2 - 60^2}{2 \times 10^3}\frac{\text{kJ}}{\text{kg}} + (3121 - 2676)\frac{\text{kJ}}{\text{kg}}\right]$$

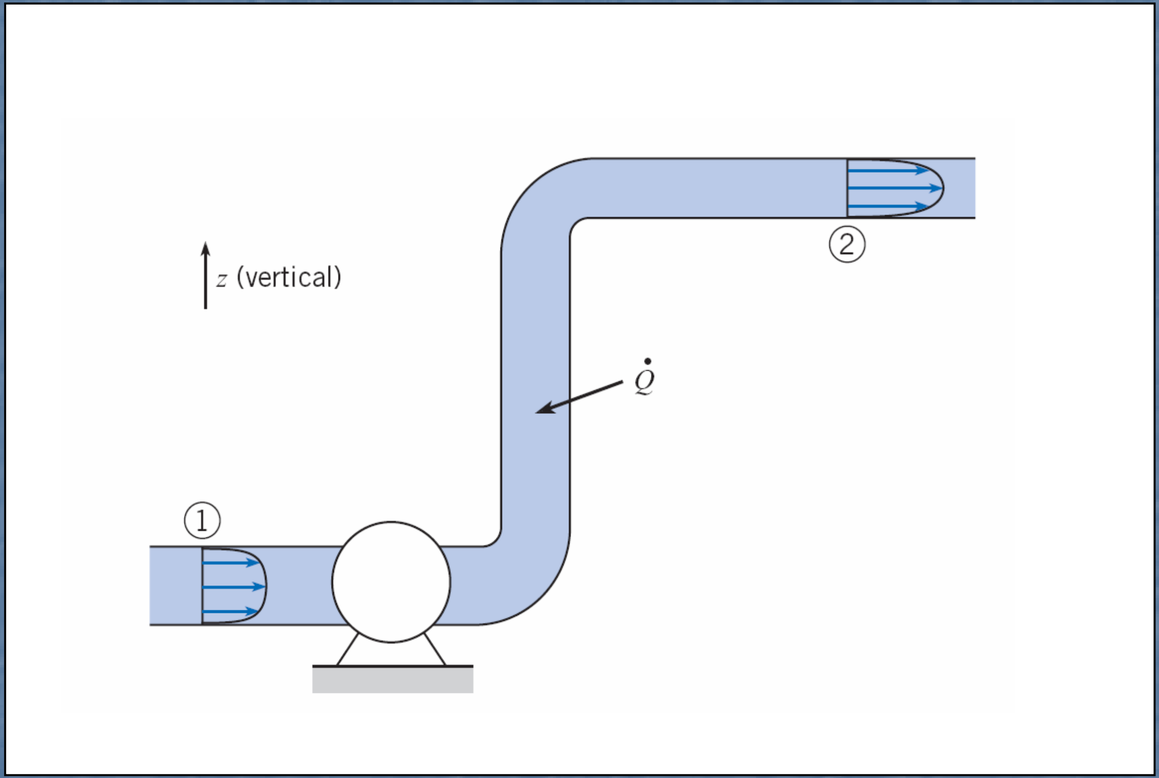
$$= -2.11 + 0.5(-1.69 + 445) = 220\frac{\text{kJ}}{\text{s}} = 220\ \text{kW}$$

$$\dot{W}_S = 220\, kW$$

$Note: (\dot{Q})\ is\ negative\ as\ heat\ flow\ out\ of\ the\ system$

ENERGY PRINCIPLE

Energy Equation for Steady Flow of an Incompressible Fluid in a pipe



Consider the flow as shown above

ENERGY PRINCIPLE

The general Energy Eqn. is written as

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{CV}(u + \frac{v^2}{2} + gz)\rho dQ + \int_{CS}(h + \frac{v^2}{2} + gz)\rho V \bullet dA$$

The energy accumulation term $\frac{d}{dt}\int_{CV}(u + \frac{v^2}{2} + gz)\rho dQ = 0$ **(Steady Flow)**

$$\dot{Q} - \dot{W}_s = \int_{CS}(h + \frac{v^2}{2} + gz)\rho V \bullet dA \quad where\ (h = \frac{p}{\rho} + u)$$

Applying the above equation between section 1 & 2, we have,

$$\dot{Q} - \dot{W}_s = \int_{A2}(h_2 + \frac{V_2^{\ 2}}{2} + gz_2)\rho V_2 dA_1 - \int_{A1}(h_1 + \frac{V_1^{\ 2}}{2} + gz_1)\rho V_1 dA_1$$

ENERGY PRINCIPLE

$$\text{The term enthalpy} = h = \left(u + \frac{p}{\rho}\right)$$

$$\dot{Q} - \dot{W}_s = \int_{A2} \left(\frac{p_2}{\rho} + u_2 + \frac{V_2^{\ 2}}{2} + gz_2\right)\rho V_2 dA_1 - \int_{A1} \left(\frac{p_1}{\rho} + u_1 + \frac{V_1^{\ 2}}{2} + gz_1\right)\rho V_1 dA_1$$

$$\dot{Q} - \dot{W}_s + \int_{A1}\left(\frac{p_1}{\rho} + gz_1 + u\right)\rho V_1 dA_1 + \int_{A1}\frac{\rho V_1^{\ 3}}{2} dA_1 = \int_{A2}\left(\frac{p_2}{\rho} + gz_2 + u_2\right)\rho V_2 dA_1 + \int_{A2}\frac{\rho V_2^{\ 3}}{2} dA_2$$

It is common to express the term $\int \frac{\rho V^3}{2} dA = \alpha\left(\frac{\overline{V}^3}{2}\right)A$

Substituting for $\int \frac{\rho V^3}{2} dA = \alpha\left(\frac{\overline{V}^3}{2}\right)A$ **in Eqn. above and dividing through by** $\dot{m}$

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}_s) + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1\frac{\overline{V}_1^{\ 2}}{2}\right) = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2\frac{\overline{V}_2^{\ 2}}{2}\right)$$

ENERGY PRINCIPLE

The coefficients (α_1, α_2) *are called kinetic energy correction factor and can be evaluated as follows :*

$$\alpha \frac{\rho \overline{V}^3 A}{2} = \int_A \frac{\rho V^3 dA}{2}$$

$$\alpha = \frac{1}{A} \int_A \left(\frac{V}{\overline{V}} \right)^3 dA$$

$\alpha = 1$ *Velocity is uniform*

$\alpha > 1$ *Velocity is non-uniform*

$\alpha = 2$ *Laminar Flow*

$\alpha = 1.05$ *Turbulent Flow*

END OF LECTURE (2)